CHAPTER 22

Vector Algebra

Exercise

1. If $ABCD$ is a rhombus whose diagonals cut at the origin O, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equals
(a) $\overrightarrow{AB}+\overrightarrow{AC}$ (b) $\vec{O}$
(c) $2(\overrightarrow{AB}+\overrightarrow{BC})$ (d) $\overrightarrow{AC}+\overrightarrow{BD}$

2. If the vectors $\vec{a}+\lambda\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$ and $\vec{a}-3\vec{b}+5\vec{c}$ are coplanar, then the value of λ is
(a) 2 (b) – 1
(c) 1 (d) – 2

3. If $\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$, then A, B, C are
(a) coplanar (b) collinear
(c) non-collinear (d) None of these

4. If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}\cdot\vec{b}\geq 0$, then
(a) $0\leq\theta\leq\pi$ (b) $\frac{\pi}{2}\leq\theta\leq\pi$
(c) $0\leq\theta\leq\frac{\pi}{2}$ (d) $0<\theta<\frac{\pi}{2}$

5. The angle between the vectors $2\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}-\hat{k}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) None of these

6. If $|\vec{a}|=|\vec{b}|=|\vec{a}+\vec{b}|=1$, then $|\vec{a}-\vec{b}|$ is equal to
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) None of these

7. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ such that $\vec{a}+\vec{b}$ is a unit vector, then θ is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

8. If $|\vec{a}|=|\vec{b}|$, then
(a) $(\vec{a}+\vec{b})$ is parallel to $\vec{a}-\vec{b}$
(b) $\vec{a}+\vec{b}\perp\vec{a}-\vec{b}$
(c) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=|\vec{a}|^2$
(d) None of the above

9. The vectors $3\hat{i}-2\hat{j}+\hat{k}, \hat{i}-3\hat{j}+5\hat{k}$ and $2\hat{i}+\hat{j}-4\hat{k}$ from the sides of a triangle, This triangle is
(a) an acute angled triangle
(b) an obtuse angled triangle
(c) a right angled triangle
(d) an equilateral triangle

10. If $\vec{a}$ and $\vec{b}$ are not perpendicular to each other and $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}, \vec{r}\cdot\vec{c}=0$, then $\vec{r}$ is equal to
(a) $\vec{a}-\vec{c}$ (b) $\vec{b}+x\vec{a}$ for all scalars
(c) $\vec{b}-\frac{(\vec{b}\cdot\vec{c})}{(\vec{a}\cdot\vec{c})}\vec{a}$ (d) None of these

11. The vectors $\vec{X}$ and $\vec{Y}$ satisfy the equations $2\vec{X}+\vec{Y}=\vec{p}$ and $\vec{X}+2\vec{Y}=\vec{q}$, where $\vec{p}=\hat{i}+\hat{j}$ and $\vec{q}=\hat{i}-\hat{j}$. If θ is the angle between $\vec{X}$ and $\vec{Y}$, then
(a) $\cos\theta=\frac{4}{5}$ (b) $\sin\theta=\frac{1}{\sqrt{2}}$
(c) $\cos\theta=-\frac{4}{5}$ (d) $\cos\theta=-\frac{3}{5}$

12. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}, \vec{a}\neq\vec{0}$, then
(a) $\vec{b}=\vec{c}$ (b) $\vec{b}-\vec{c}\parallel\vec{a}$
(c) $\vec{b}-\vec{c}\perp\vec{a}$ (d) None of the above

13. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=0=\vec{a}\cdot\vec{c}$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then $\vec{a}$ equals
(a) $\pm 2(\vec{b}+\vec{c})$ (b) $2(\vec{b}+\vec{c})$
(c) $\pm\frac{1}{2}(\vec{b}\times\vec{c})$ (d) $-\frac{1}{2}(\vec{b}\times\vec{c})$

14. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then
(a) $(\vec{a}-\vec{d})=\lambda(\vec{b}-\vec{c})$ (b) $(\vec{a}-\vec{d})=\lambda(\vec{b}+\vec{c})$
(c) $(\vec{a}-\vec{b})=\lambda(\vec{c}+\vec{d})$ (d) None of these

15. If $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b}$ and $|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is
(a) $2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$ (b) $2\sqrt{4-(\vec{a}\cdot\vec{b})^2}$
(c) $\sqrt{16-(\vec{a}\cdot\vec{b})^2}$ (d) $\sqrt{4-(\vec{a}\cdot\vec{b})^2}$

16. Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ be a unit vector $\perp$ to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then it is given by
(a) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$
(c) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\frac{1}{2}(\hat{j}-\hat{k})$

17. If the vectors $\vec{a}=\hat{i}+a\hat{j}+a^2\hat{k}, \vec{b}=\hat{i}+b\hat{j}+b^2\hat{k}, \vec{c}=\hat{i}+c\hat{j}+c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0,$ then the value of abc is
(a) 0 (b) 1
(c) 2 (d) –1

18. For any three vectors $\vec{a}, \vec{b}, \vec{c}, (\vec{b}\times\vec{c})\times\vec{a}$ equals
(a) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$ (b) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$
(c) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{c}-\vec{a})\vec{b}$ (d) None of these

19. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_2\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector $\perp$ to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to
(a) 0 (b) 1
(c) $\frac{1}{4}|\vec{a}||\vec{b}|^2$ (d) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

20. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$ is
(a) $\vec{0}$ (b) $[\vec{a}\vec{b}\vec{c}]$
(c) $3[\vec{a}\vec{b}\vec{c}]$ (d) $[\vec{a}\vec{b}\vec{c}]$

21. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}=\vec{c}$, then
(a) $|\vec{a}|^2+|\vec{b}|^2=|\vec{c}|^2$ (b) $|\vec{a}|^2=|\vec{b}|^2+|\vec{c}|^2$
(c) $|\vec{b}|^2=|\vec{a}|^2+|\vec{c}|^2$ (d) None of these

22. The scalar $\vec{a}\cdot\{(\vec{b}+\vec{a})\times(\vec{a}+\vec{b}+\vec{c})\}$ equals
(a) 0 (b) $2[\vec{a}\vec{b}\vec{c}]$
(c) $[\vec{a}\vec{b}\vec{c}]$ (d) None of these

23. $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})\cdot\vec{d}$ equals to
(a) $[\vec{a}\vec{b}\vec{c}](\vec{b}\cdot\vec{d})$ (b) $[\vec{a}\vec{b}\vec{c}](\vec{a}\cdot\vec{d})$
(c) $[\vec{a}\vec{b}\vec{c}](\vec{c}\cdot\vec{d})$ (d) None of these

24. If $\vec{a}$ is a vector of magnitude 50 collinear with the vector $\vec{b}=6\hat{i}-8\hat{j}-\frac{15}{2}\hat{k}$ and makes an acute angle with the positive direction of z-axis, then $\vec{a}$ is
(a) $24\hat{i}-32\hat{j}-30\hat{k}$ (b) $-24\hat{i}+32\hat{j}+30\hat{k}$
(c) $12\hat{i}-16\hat{j}-15\hat{k}$ (d) None of these

25. The projection of the vectors $\vec{a}=4\hat{i}-3\hat{j}+2\hat{k}$ on the axis making equal acute angles with the coordinate axes is
(a) 3 (b) $\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) None of these

26. If forces of magnitudes 6 and 7 units acting in the directions $\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}-3\hat{j}-6\hat{k}$ respectively act on a particle which is displaced from the point $P(2,-1,-3)$ to $Q(5,-1,1)$, then the work done by the forces is
(a) 4 units (b) – 4 units
(c) 7 units (d) – 7 units

27. If $\vec{a}=-2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+5\hat{j}$ and $\vec{c}=4\hat{i}+4\hat{j}-2\hat{k}$, then projection of $3\vec{a}-2\vec{b}$ on the axis of the vector $\vec{c}$ is
(a) 11 (b) – 11
(c) – 12 (d) – 33

28. If the force represented by $3\hat{j}+2\hat{k}$ is acting through the point $5\hat{i}+4\hat{j}-3\hat{k}$, then its moment about the point (1, 3, 1) is

(a) $14\hat{i}-8\hat{j}+12\hat{k}$ (b) $-14\hat{i}+8\hat{j}-12\hat{k}$

(c) $42\hat{i}+144\hat{j}-24\hat{k}$ (d) None of these

29. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $m=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, then

(a) $m<0$ (b) $m>0$

(c) $m=0$ (d) $m=3$

30. If $\vec{a},\vec{b},\vec{c}$ non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\dfrac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\dfrac{3\pi}{4}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{2}$ (d) π

31. The vector $\vec{c}$ is perpendicular to the vectors $\vec{a}=(2-3,1)$, $\vec{b}=(1-2,3)$, and satisfies the conditions $\vec{c}\cdot(\hat{i}+2\hat{j}-7\hat{k})=10$. Then, $\vec{c}$ is

(a) $7\hat{i}+5\hat{j}+\hat{k}$ (b) $-7\hat{i}-5\hat{j}-\hat{k}$

(c) $\hat{i}+\hat{j}-\hat{k}$ (d) None of these

32. If the vectors $2\hat{i}-3\hat{j}+4\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$ and $x\hat{i}-\hat{j}+2\hat{k}$ are coplanar, then x is

(a) $\dfrac{8}{5}$ (b) $\dfrac{5}{8}$

(c) 0 (d) 1

33. The value of b such that the scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $b\hat{i}+2\hat{j}+3\hat{k}$ is one, is

(a) – 2 (b) –1

(c) 0 (d) 1

34. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is

(a) $2\hat{i}+3\hat{j}-3\hat{k}$

(b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-5\hat{j}+5\hat{k}$

(d) $2\hat{i}+\hat{j}+5\hat{k}$

35. A unit vector perpendicular to $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$ is

(a) $\dfrac{1}{3}(\hat{i}-2\hat{j}+2\hat{k})$ (b) $\dfrac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k})$

(c) $\dfrac{1}{3}(2\hat{i}+\hat{j}+2\hat{k})$ (d) $\dfrac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

36. Given that $\vec{a}=(1,1,1)$, $\vec{c}=(0,1,-1)$ and $\vec{a}\cdot\vec{b}=3$. If $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

(a) $\left(\dfrac{1}{2},-\dfrac{1}{2},\dfrac{1}{2}\right)$ (b) $\left(\dfrac{2}{3},\dfrac{2}{3},\dfrac{4}{3}\right)$

(c) $\left(\dfrac{5}{3},\dfrac{2}{3},\dfrac{2}{3}\right)$ (d) None of these

37. If $|\vec{a}|=4$, $|\vec{b}|=4$ and $|\vec{c}|=5$ such that $\vec{a}\perp(\vec{b}+\vec{c})$, $\vec{b}\perp(\vec{c}+\vec{a})$ and $\vec{c}\perp(\vec{a}+\vec{b})$, then $|\vec{a}+\vec{b}+\vec{c}|$ is

(a) – 7 (b) 5

(c) – 13 (d) $\sqrt{57}$

38. The number of vectors of unit length perpendicular to vectors $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$ is

(a) one (b) two

(c) three (d) None

39. If $\vec{a}$ and $\vec{b}$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of a regular hexagon *ABCDEF*, then $\overrightarrow{FA}$ is

(a) $\vec{b}-\vec{a}$ (b) $\vec{a}-\vec{b}$

(c) $\vec{a}+\vec{b}$ (d) None of these

40. Let $\vec{a},\vec{b},\vec{c}$ be three vectors ($\neq 0$), no two of which are collinear. If $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ then $\vec{a}+2\vec{b}+6\vec{c}$ is

(a) parallel to $\vec{a}$ (b) parallel to $\vec{b}$

(c) parallel to $\vec{c}$ (d) 0

41. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a}+\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$

(c) $\vec{c}$ (d) 0

42. The point with position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear if a equals

(a) –8 (b) 4

(c) 8 (d) 12

43. If G is the centroid of a triangle ABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ equals

(a) $\vec{0}$ (b) $3\overrightarrow{GA}$

(c) $3\overrightarrow{GB}$ (d) $3\overrightarrow{GC}$

44. If $\vec{a}$ and $\vec{b}$ are position vectors of A and B respectively the position vector of a point C on AB produced such that $\overrightarrow{AC} = 3\overrightarrow{AB}$ is

(a) $3\vec{a}-\vec{b}$ (b) $3\vec{b}-\vec{a}$
(c) $3\vec{a}-2\vec{b}$ (d) $3\vec{b}-2\vec{a}$

45. Find the horizontal force and a force inclined at an angle of 60° with the vertical so that the resultant is a vertical force of P kg wt,

(a) $P, 2P$ (b) $P, P\sqrt{3}$
(c) $P\sqrt{3}, 2P$ (d) None of these

46. If the resultant of two forces is of magnitude P and equal to one of them and perpendicular to it, then the other force is

(a) P (b) $P\sqrt{3}$
(c) $P\sqrt{2}$ (d) $2P\sqrt{3}$

47. The position vectors of the points A and B with respect to an origin are $\vec{a}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$ respectively. If P is point on AB, then vector $\overrightarrow{OP}$ which bisects $\angle AOB$ is

(a) $2(-\hat{i}+\hat{j}+\hat{k})$ (b) $2(\hat{i}-\hat{j}+\hat{k})$
(c) $2(\hat{i}+\hat{j}-\hat{k})$ (d) $2(\hat{i}+\hat{j}+\hat{k})$

48. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(a) $6(\vec{b}\times\vec{c})$ (b) $2(\vec{a}\times\vec{b})$
(c) $3(\vec{c}\times\vec{a})$ (d) All are correct

49. The vectors $\vec{a},\vec{b},\vec{c}$ are equal in length and taken pairwise they make equal angles. If $\vec{a}=\hat{i}+\hat{j}, \vec{b}=\hat{j}+\hat{k}$ and $\vec{c}$ makes obtuse angle with X-axis, then $\vec{c}$ is

(a) –1, 4, –1 (b) 1, 0, 1
(c) –1/3, 4/3, –1/3 (d) 1/3, –4/3, 1/3

50. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then the angle θ which $\vec{a}+\vec{b}+\vec{c}$ makes with any one of three given vectors is given by

(a) $\cos^{-1}(1/\sqrt{3})$ (b) $\cos^{-1}(1/3)$
(c) $\cos^{-1}(2/\sqrt{3})$ (d) None of these

51. Let $\vec{u},\vec{v},\vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2$ and $|\vec{w}|=3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v},\vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ is

(a) – 2 (b) $\sqrt{7}$
(c) $\sqrt{14}$ (d) 14

52. The vectors $2\hat{i}-m\hat{j}+3m\hat{k}$ and $(1+m)\hat{i}-2m\hat{j}+\hat{k}$ include an acute angle for

(a) all values of m (b) $m<-2$
(c) $m>\frac{1}{2}$ (d) $m\in\left[-2,-\frac{1}{2}\right]$

53. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter-clockwise sense. If with respect to new system, $\vec{a}$ has components $p+1$ and 1, then

(a) $p=0$ (b) $p=1$ or $p=-\frac{1}{3}$
(c) $p=-1$ or $p=\frac{1}{3}$ (d) $p=1$ or $p=-1$

54. The vector $\vec{a}+3\vec{b}$ is perpendicular to $7\vec{a}-5\vec{b}$ and $\vec{a}-5\vec{b}$ is perpendicular to $7\vec{a}+3\vec{a}$. The angle between $\vec{a}$ and $\vec{b}$ is

(a) π / 4 (b) π/6
(c) π/2 (d) None of these

55. The position vectors of four points A, B, C, D lying in a plane are $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively. They satisfy the relation $|\vec{a}-\vec{d}|=|\vec{b}-\vec{d}|=|\vec{c}-\vec{d}|$, then the point D is

(a) centroid of ΔABC (b) circumcentre of ΔABC
(c) orthocentre of ΔABC (d) incentre of ΔABC

56. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of point A, B, C and D respectively such that : $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$, then D is the

(a) centroid of ΔABC (b) circumcentre of ΔABC
(c) orthocentre of ΔABC (d) None of these

57. The vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$ are adjacent sides of parallelogram, then angle between its diagonals is

(a) π/4 (b) π/3
(c) 3π/4 (d) 2π/3

58. The resultant of two forces PN and $3N$ is a force of $7N$. If the direction of $3N$ force were reversed, the resultant would be $\sqrt{19}N$. The value of P is

(a) $5N$ (b) $6N$
(c) $3N$ (d) $4N$

59. Let $\vec{a}=3i+2k$ and $\vec{b}=2j+k$. If $\vec{c}$ is a unit vector, then the maximum value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is

(a) $\sqrt{59}$ (b) $\sqrt{61}$
(c) $\sqrt{108}$ (d) None of these

60. Let $\vec{b}=-\hat{i}+4\hat{j}+6\hat{k}, \vec{c}=2\hat{i}-7\hat{j}-10\hat{k}$. If $\vec{a}$ be a unit vector and the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ has the

greatest value, then $\vec{a}$ is

(a) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{5}}(\sqrt{2}\hat{i}-\hat{j}-\sqrt{2}\hat{k})$

(c) $\frac{1}{3}(2\hat{i}+2\hat{j}-\hat{k})$ (d) $\frac{1}{\sqrt{5}}(2\hat{i}-7\hat{j}-\hat{k})$

61. The vector moment of three forces $\hat{i}+2\hat{j}-3\hat{k}, 2\hat{i}+3\hat{j}+4\hat{k}$ and $-\hat{i}-\hat{j}-\hat{k}$ acting on a particle at a point $P(0, 1, 2)$ about the point $A(1, -2, 0)$ is

(a) $-2(4\hat{i}-2\hat{j}+5\hat{k})$ (b) $4\hat{i}+5\hat{j}+6\hat{k}$

(c) $7\hat{i}+2\hat{k}$ (d) None of these

62. What is the area of the parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$? **[NDA-I 2016]**

(a) $5\sqrt{2}$ sq. units (b) $4\sqrt{5}$ sq. units

(c) $5\sqrt{3}$ sq. units (d) $15\sqrt{2}$ sq. units

63. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\vec{a}-\vec{b}}{2}\right|$ is **[NDA-I 2016]**

(a) $\sin\frac{\theta}{2}$ (b) $\sin\theta$

(c) $2\sin\theta$ (d) $\sin 2\theta$

64. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct? **[NDA-I 2016]**

(a) $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (b) $\vec{a}+\vec{b}+\vec{c}=$ unit vector

(c) $\vec{a}+\vec{b}=\vec{c}$ (d) $\vec{a}=\vec{b}+\vec{c}$

65. If C is the middle point of AB and P is any point outside AB, then

(a) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (b) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$

(c) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$

66. If $\hat{a}$ and $\hat{b}$ are the two unit vectors and θ be the angle between them, then what is $\cos\frac{\theta}{2}$ equal to **[NDA-I 2016]**

(a) $\frac{|\hat{a}-\hat{b}|}{2}$ (b) $\frac{|\hat{a}+\hat{b}|}{2}$

(c) $\frac{|\hat{a}-\hat{b}|}{4}$ (d) $\frac{|\hat{a}+\hat{b}|}{4}$

67. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}, |\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is **[NDA-II 2016]**

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{3}$

68. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=2$, $|\vec{b}|=3, |\vec{c}|=4$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to **[NDA-II 2016]**

(a) 29 (b) –29

(c) $\frac{29}{2}$ (d) $-\frac{29}{2}$

69. A force $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point (1, –1, 2). What is the moment of the force about the point (2, –1, 3)? **[NDA-II 2016]**

(a) $\hat{i}+4\hat{j}+4\hat{k}$ (b) $2\hat{i}+\hat{j}+2\hat{k}$

(c) $2\hat{i}-7\hat{j}-2\hat{k}$ (d) $2\hat{i}+4\hat{j}-\hat{k}$

Directions (Q. Nos. 70 and 71) :

Let $\vec{a}=\hat{i}+\hat{j}, \vec{b}=3\hat{i}+4\hat{k}$ and $\vec{b}=\vec{c}+\vec{d}$, where $\vec{c}$ is parallel to $\vec{a}$ and $\vec{d}$ is perpendicular to $\vec{a}$.

70. What is $\vec{c}$ is equal to? **[NDA-II 2016]**

(a) $\frac{3(\hat{i}+\hat{j})}{2}$ (b) $\frac{2(\hat{i}+\hat{j})}{3}$

(c) $\frac{(\hat{i}+\hat{j})}{3}$ (d) $\frac{(\hat{i}+\hat{j})}{2}$

71. If $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$, then which of the following equation is/are correct? **[NDA-II 2016]**

1. $y-x=4$ 2. $2z-3=0$

Select the correct answer using the code given below:

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

72. If in right angled triangle ABC, the hypotenuse $AB=p$, then $\overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BC}\cdot\overrightarrow{BA}+\overrightarrow{CA}\cdot\overrightarrow{CB}$ is equal to **[NDA-II 2016]**

(a) $2p^2$ (b) $p^2/2$

(c) p^2 (d) p

73. Let $ABCD$ be a parallelogram whose diagonals intersect at P and let O be the origin, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equals **[NDA-I 2017]**

(a) $\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$

(c) $3\overrightarrow{OP}$ (d) $4\overrightarrow{OP}$

74. If $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-\lambda\hat{k}$ are perpendicular, then what is the value of λ ? **[NDA-I 2017]**

(a) 2 (b) 3

(c) 4 (d) 5

75. $ABCD$ is a quadrilateral whose diagonals are AC and BD. Which one of the following is correct?

[NDA-I 2017]

(a) $\overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{AC}+\overrightarrow{DB}$ (b) $\overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{BD}+\overrightarrow{CA}$

(c) $\overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{AC}+\overrightarrow{BD}$ (d) $\overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{BC}+\overrightarrow{AD}$

76. If $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, then which one of the following is correct? **[NDA-I 2017]**

(a) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs and $|\vec{a}|=|\vec{c}|$ and $|\vec{b}|=1$

(b) $\vec{a},\vec{b},\vec{c}$ are non-orthogonal to each other

(c) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs but $|\vec{a}|\neq|\vec{c}|$

(d) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs but $|\vec{b}|\neq 1$

77. If $\vec{a}=\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{c}=\hat{i}+m\hat{j}+n\hat{k}$ are three coplanar vectors and $|\vec{c}|=\sqrt{6}$, then which one of the following is correct? **[NDA-I 2017]**

(a) $m=2$ and $n=\pm 1$

(b) $m=\pm 2$ and $n=-1$

(c) $m=2$ and $n=-1$

(d) $m=\pm 2$ and $n=1$

78. For any vector $\vec{a}$, $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$ is equal to **[NDA-II 2017]**

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$

(c) $3|\vec{a}|^2$ (d) $4|\vec{a}|^2$

79. If $\hat{a}$ and $\hat{b}$ are two unit vectors, then the vector $(\hat{a}+\hat{b})\times(\hat{a}\times\hat{b})$ is parallel to **[NDA-II 2017]**

(a) $(\hat{a}-\hat{b})$ (b) $(\hat{a}+\hat{b})$

(c) $(2\hat{a}-\hat{b})$ (d) $(2\hat{a}+\hat{b})$

80. Let $\vec{\alpha}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{\gamma}=2\hat{i}+\hat{j}+6\hat{k}$ be three vectors. If $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to the vector $\vec{\delta}$ and $\vec{\delta}\cdot\vec{\gamma}=10$, then what is the magnitude of $\vec{\delta}$? **[NDA-II 2017]**

(a) $\sqrt{3}$ units (b) $2\sqrt{3}$ units

(c) $\dfrac{\sqrt{3}}{2}$ units (d) $\dfrac{1}{\sqrt{3}}$ units

ANSWERS

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(b)	**9.**	(c)	**10.**	(c)
11.	(c)	**12.**	(a)	**13.**	(a)	**14.**	(a)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(b)	**19.**	(c)	**20.**	(a)
21.	(a)	**22.**	(c)	**23.**	(b)	**24.**	(b)	**25.**	(b)	**26.**	(a)	**27.**	(b)	**28.**	(a)	**29.**	(a)	**30.**	(a)
31.	(a)	**32.**	(a)	**33.**	(d)	**34.**	(a)	**35.**	(b)	**36.**	(c)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(d)
41.	(d)	**42.**	(c)	**43.**	(a)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(c)	**48.**	(d)	**49.**	(c)	**50.**	(c)
51.	(c)	**52.**	(b)	**53.**	(b)	**54.**	(c)	**55.**	(b)	**56.**	(c)	**57.**	(a)	**58.**	(a)	**59.**	(b)	**60.**	(c)
61.	(a)	**62.**	(c)	**63.**	(a)	**64.**	(a)	**65.**	(a)	**66.**	(b)	**67.**	(d)	**68.**	(d)	**69.**	(c)	**70.**	(a)
71.	(d)	**72.**	(c)	**73.**	(d)	**74.**	(b)	**75.**	(b)	**76.**	(a)	**77.**	(d)	**78.**	(b)	**79.**	(a)	**80.**	(b)

Explanations

1. (b) $\because$ Diagonals of the rhombus bisect each other.

So, $\overrightarrow{OA}=-\overrightarrow{OC}$ and $\overrightarrow{OB}=-\overrightarrow{OD}$

$\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OC}=\vec{O}$

2. (d) $\overrightarrow{OA}=\vec{a}+\lambda\vec{b}+3\vec{c}$, $\overrightarrow{OB}=-2\vec{a}+3\vec{b}-4\vec{c}$,

$\overrightarrow{OC}=\vec{a}-3\vec{b}+5\vec{c}$ are coplanar.

So, scalar triple product = 0

$[\overrightarrow{OA}\,\overrightarrow{OB}\,\overrightarrow{OC}]=0$

$$\Rightarrow \begin{vmatrix} 1 & \lambda & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}=0$$

$\Rightarrow (15-12)-\lambda(-10+4)+3(6-3)=0$

$\Rightarrow 3+6\lambda+9=0 \Rightarrow 6\lambda=-12$

$\Rightarrow \lambda=-2$

3. (b) $\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$

$\Rightarrow \overrightarrow{AB}=\overrightarrow{BC}$

This is the condition of collinear.

4. (c) $\because \vec{a}\cdot\vec{b}\geq 0$

$\Rightarrow ab\cos\theta\geq 0 \Rightarrow \cos\theta\geq 0$

$\Rightarrow 0\leq\theta\leq\dfrac{\pi}{2}$

5. (b) $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}, \vec{b}=2\hat{i}-\hat{j}-\hat{k}$

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{4-3-1}{\sqrt{4+9+1}\sqrt{4+1+1}}$$

$$\cos\theta=0=\cos 90^\circ \Rightarrow \theta=90^\circ=\frac{\pi}{2}$$

6. (c) $|\vec{a}|=|\vec{b}|=|\vec{a}+\vec{b}|=1$

Then, $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2(a^2+b^2)$

$\Rightarrow 1+|\vec{a}-\vec{b}|^2=2(1+1)$

$\Rightarrow |\vec{a}-\vec{b}|^2=4-1=3 \Rightarrow |\vec{a}-\vec{b}|=\sqrt{3}$

7. (d) $|\vec{a}|=|\vec{b}|=|\vec{a}+\vec{b}|=1$

$\because |\vec{a}+\vec{b}|^2=a^2+b^2+2\vec{a}\cdot\vec{b}$

$\Rightarrow 1=1+1+2ab\cos\theta$

$\Rightarrow 1=2+2\cos\theta$

$\Rightarrow \cos\theta=-\frac{1}{2}\Rightarrow \cos\theta=\cos\frac{2\pi}{3}\Rightarrow \theta=\frac{2\pi}{3}$

8. (b) $(\vec{a}+\vec{b})\perp(\vec{a}-\vec{b})$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$

$\Rightarrow \vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=0$

$\Rightarrow |\vec{a}|^2=|\vec{b}|^2\Rightarrow |\vec{a}|=|\vec{b}|$

9. (c) $\overrightarrow{AB}=3\vec{i}-2\vec{j}+\vec{k}$,

$\Rightarrow |\overrightarrow{AB}|=\sqrt{9+4+1}=\sqrt{14}$;

$\overrightarrow{BC}=\hat{i}-3\hat{j}+5\hat{k}$

$\Rightarrow |\overrightarrow{BC}|=\sqrt{1+9+25}=\sqrt{35}$

and $\overrightarrow{CA}=2\hat{i}+\hat{j}-4\hat{k}$

$\Rightarrow |\overrightarrow{CA}|=\sqrt{4+1+16}=\sqrt{21}$

So, $|\overrightarrow{BC}|^2=|\overrightarrow{BC}|^2+|\overrightarrow{CA}|^2$

$\Rightarrow$ Right angled triangle.

10. (c) $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$

$\Rightarrow (\vec{r}\times\vec{a})-(\vec{b}\times\vec{a})=0$

$\Rightarrow (\vec{r}-\vec{b})\times\vec{a}=0\Rightarrow(\vec{r}-\vec{b})\parallel\vec{a}$

$\Rightarrow \vec{r}-\vec{b}=t\vec{a}$, where t is scalar.

or $\vec{r}=\vec{b}+t\vec{a}$ ...(i)

Now, $\vec{r}\cdot\vec{c}=0$ (given)

$(\vec{b}+t\vec{a})\cdot\vec{c}=0$

$\vec{b}\cdot\vec{c}+t\vec{a}\cdot\vec{c}=0$

$t=-\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{c}}$ Put in eq. (i)

$$\vec{r}=\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{c}}\right)\vec{a}$$

11. (c) $2\vec{X}+\vec{Y}=\vec{p}, \vec{X}+2\vec{Y}=\vec{q}$

$\Rightarrow 2\vec{X}+\vec{Y}=\hat{i}+\hat{j}$,

and $\vec{X}+2\vec{Y}=\hat{i}-\hat{j}$ $\{\because \vec{p}=\hat{i}+\hat{j}$ and $\vec{q}=\hat{i}-\hat{j}\}$

On solving both equations,

$-3\vec{Y}=-\hat{i}+3\hat{j}$

$\vec{Y}=\frac{\hat{i}}{3}-\hat{j}$

$\vec{X}=(\hat{i}-\hat{j})-2\vec{Y}$

$=(\hat{i}-\hat{j})-2\left(\frac{\hat{i}}{3}-\hat{j}\right)$

$\vec{X}=\frac{\hat{i}}{3}+\hat{j}$

$$\cos\theta=\frac{\vec{X}\cdot\vec{Y}}{|\vec{X}||\vec{Y}|}=\frac{\frac{1}{9}-1}{\sqrt{\frac{1}{9}+1}\sqrt{\frac{1}{9}+1}}=\frac{-\frac{8}{9}}{\frac{10}{9}}$$

$$\cos\theta=-\frac{4}{5}$$

12. (a) $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}\Rightarrow\vec{a}\cdot(\vec{b}-\vec{c})=0$

$\Rightarrow \vec{a}\perp(\vec{b}-\vec{c})$ or $\vec{a}=0$ or $\vec{b}=\vec{c}$ …(i)

$\vec{a}\times\vec{b}=\vec{a}\times\vec{c}\Rightarrow\vec{a}\times(\vec{b}-\vec{c})=0$

$\Rightarrow \vec{a}\parallel\vec{b}-\vec{c}$ *or* $\vec{a}=0$ *or* $\vec{b}=\vec{c}$ …(ii)

From eqs. (i) and (ii)

$\vec{b}=\vec{c}$ $\{\because \vec{a}\neq 0$ given$\}$

13. (a) $\because$ Given $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$

$\Rightarrow \vec{a}$ is $\perp$ to both $\vec{b}$ and $\vec{c}$

Hence, $\vec{a}=\pm\frac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|}$

$$\vec{a}=\pm\frac{\vec{b}\times\vec{c}}{bc\sin\frac{\pi}{6}}=\pm\frac{2(\vec{b}\times\vec{c})}{1}$$

$\vec{a}=\pm 2(\vec{b}\times\vec{c})$

14. (a) $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ …(i)

$\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ …(ii)

From eq. (i) – (ii),

$\vec{a}\times(\vec{b}-\vec{c})=(\vec{c}-\vec{b})\times\vec{d}$

or $\vec{a}\times(\vec{b}-\vec{c})-(\vec{c}-\vec{b})\times\vec{d}=0$

or $\vec{a}\times(\vec{b}-\vec{c})+\vec{d}\times(\vec{c}-\vec{b})=0$

$(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=0$

$\Rightarrow \vec{a}-\vec{d} \parallel \vec{b}-\vec{c}$ or $\vec{a}-\vec{d}=\lambda(\vec{b}-\vec{c})$

where, λ is any constant.

15. (a) $\vec{u}=\vec{a}-\vec{b},\ \vec{v}=\vec{a}+\vec{b},\ |\vec{a}|=|\vec{b}|=2$

$|\vec{u}\times\vec{v}|=|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|$

$=|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}+\vec{b}\times\vec{b}|$

$=|2(\vec{a}\times\vec{b})|=2|\vec{a}\times\vec{b}|$

$=2\sqrt{a^2b^2-(\vec{a}\cdot\vec{b})^2}=2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$

16. (a) $\vec{a}=(1,\ 1,\ -1),\ \vec{b}=(1,\ -1,\ 1)$

Vector coplanar with $\vec{a}$ and $\vec{b}$ and $\perp$ to $\vec{a}$ is

$\vec{r}=\vec{a}\times(\vec{a}\times\vec{b})=(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}$

$=(1-1-1)(\hat{i}+\hat{j}-\hat{k})-(\sqrt{1+1+1})^2(\hat{i}-\hat{j}+\hat{k})$

$=-\hat{i}-\hat{j}+\hat{k}-3\hat{i}+3\hat{j}-3\hat{k}$

$\vec{r}=-4\hat{i}+2\hat{j}-2\hat{k}$

Unit vector $=\pm\dfrac{\vec{r}}{|\vec{r}|}$

$=\pm\dfrac{-4\hat{i}+2\hat{j}-2\hat{k}}{\sqrt{16+4+4}}$

$=\pm\dfrac{1}{\sqrt{6}}(-2\hat{i}+\hat{j}-\hat{k})$

17. (d) $\vec{a},\vec{b},\vec{c}$ are non-coplanar, so $[\vec{a},\vec{b},\vec{c}]\neq 0$

$$\Rightarrow \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0\Rightarrow \Delta\neq 0$$

Now, given $\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$$

$\Rightarrow\ \Delta(1+abc)=0$

$\Delta\neq 0\Rightarrow 1+abc=0$

$\Rightarrow\ abc=-1$

18. (b) $(\vec{b}\times\vec{c})\times\vec{a}=-\vec{a}\times(\vec{b}\times\vec{c})$

$=-(\vec{a}\cdot\vec{c})\vec{b}+(\vec{a}\cdot\vec{b})\vec{c}$

$=(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$

19. (c) $\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}^2=[\vec{a}\ \vec{b}\ \vec{c}]^2$

$=\{(\vec{a}\times\vec{b})\cdot\vec{c}\}^2=|\vec{a}\times\vec{b}|^2\cdot\{\vec{c}\}^2$

$=a^2b^2\sin^2\dfrac{\pi}{6}\cdot c^2=\dfrac{1}{4}|a|^2|b|^2$

20. (a) $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}(\vec{a}\times\vec{b})$

$=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}+(\vec{b}\cdot\vec{a})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$
$+(\vec{c}\cdot\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}$

$=0$

21. (a) $|\vec{a}+\vec{b}|^2=|\vec{c}|^2$

$\Rightarrow\ a^2+b^2+2a^2\cdot b^2=c^2$

$\Rightarrow\ |\vec{a}|^2+|\vec{b}|^2=|\vec{c}|^2$

22. (c) $\vec{a}\cdot\{(\vec{b}+\vec{a})\times(\vec{a}+\vec{b}+\vec{c})\}$

$=\vec{a}\cdot\{\vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c}+\vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}\}$

$=\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{a}\times\vec{c})$

$=[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{a}\,\vec{a}\,\vec{c}]=[\vec{a}\,\vec{b}\,\vec{c}]\ \{\because [\vec{a}\,\vec{a}\,\vec{c}]=0\}$

23. (b) $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})\cdot\vec{d}$

$=\{((\vec{a}\times\vec{b})\cdot\vec{c})\vec{a}-((\vec{a}\times\vec{b})\cdot\vec{a})\vec{c})\cdot\vec{d}\}$

$=\{[\vec{a}\vec{b}\vec{c}]\vec{a}-[\vec{a}\vec{b}\vec{a}]\vec{c}\}\cdot\vec{d}$

$=[\vec{a}\,\vec{b}\,\vec{c}](\vec{a}\cdot\vec{d})$

24. (b) $\vec{a}$ is collinear with $\vec{b}$

$\Rightarrow \vec{a}=t\vec{b}$

So, $\vec{a}=t\left(6\hat{i}-8\hat{j}-\dfrac{15}{2}\hat{k}\right)$

Given, $|\vec{a}|=50$

$\Rightarrow t\sqrt{36+64+\dfrac{225}{4}}=50$

$t\sqrt{\dfrac{625}{4}}=50\Rightarrow \pm t\dfrac{25}{2}=50$

$\Rightarrow\ t=\pm 4\Rightarrow \vec{a}=\pm 4\left(6\hat{i}-8\hat{j}-\dfrac{15}{2}\hat{k}\right)$

$\because\ \vec{a}$ makes an acute angle with (+) ve direction of Z-axis.

i.e., $\vec{a}\cdot\hat{k}>0\Rightarrow t=-4$

So, $a=-4\left(6\hat{i}-8\hat{j}-\dfrac{15}{2}\hat{k}\right)$

or $\vec{a}=-24\hat{i}+32\hat{j}+30\hat{k}$

25. (b) Direction cosines of a vector making equal angles with coordinate axis are

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}.$

Hence, $\vec{b} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$\vec{a} = 4\hat{i} - 3\hat{j} + 2\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{\frac{4}{\sqrt{3}} - \frac{3}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

26. (a) $\vec{F_1} = 6\frac{(\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} = 2(\hat{i} - 2\hat{j} + 2\hat{k})$

$\vec{F_2} = \frac{7(2\hat{i} - 3\hat{j} - 6\hat{k})}{\sqrt{4+9+36}} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

$\vec{F} = \vec{F_1} + \vec{F_2} = 4\hat{i} - 7\hat{j} - 2\hat{k}$

$\vec{s} = \overrightarrow{OQ} - \overrightarrow{OP} = (5\hat{i} - \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - 3\hat{k})$

$\vec{s} = 3\hat{i} + 4\hat{k}$

$\vec{W} = \vec{F}\cdot\vec{S} \Rightarrow \vec{W} = 12 - 8 = 4$ units.

27. (b) $\vec{a} = (-2, 1, 1), \vec{b} = (1, 5, 0), \vec{c} = (4, 4, -2)$

$3\vec{a} - 2\vec{b} = 3(-2, 1, 1) - 2(1, 5, 0)$

$= (-8, -7, 3)$

Projection of $(3\vec{a} - 2\vec{b})$ on $\vec{c}$

$$= \frac{(3\vec{a} - 2\vec{b})\cdot\vec{c}}{|\vec{c}|} = \frac{-32 - 28 - 6}{\sqrt{16+16+4}}$$

$$= \frac{-66}{6} = 11$$

28. (a) $\vec{F} = 3\hat{j} + 2\hat{k}$ and $\vec{r} = \overrightarrow{OP}$

$\Rightarrow \vec{r} = (5\hat{i} + 4\hat{j} - 4\hat{k} - \hat{i} - 3\hat{i} - \hat{k})$

$\vec{r} = 4\hat{i} + \hat{j} - 4\hat{k}$

$\vec{M} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -4 \\ 0 & 3 & 2 \end{vmatrix} = 14\hat{i} - 8\hat{j} + 12\hat{k}$$

29. (a) $|\vec{a} + \vec{b} + \vec{c}|^2 = a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$0 = a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{1}{2}(a^2 + b^2 + c^2)$

$\Rightarrow m = -\frac{1}{2}(a^2 + b^2 + c^2) \Rightarrow m < 0$

30. (a) $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$

On equating,

$\vec{a}\cdot\vec{c} = \frac{1}{\sqrt{2}}$ and $\vec{a}\cdot\vec{c} = -\frac{1}{\sqrt{2}}$

$\Rightarrow ab\cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} = \cos 135°$

$\Rightarrow \theta = 135° = \frac{3\pi}{4}$

31. (a) $\vec{c} \perp \vec{a}$ and $\vec{c} \perp \vec{b} \Rightarrow \vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{c} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$\vec{c} = \lambda(-7\hat{i} - 5\hat{j} - \hat{k})$

Now, $\vec{c}\cdot(\hat{i} + 2\hat{j} - 7\hat{k}) = 10$

$\Rightarrow \lambda = -1$

So, $\vec{c} = 7\hat{i} + 5\hat{j} + \hat{k}$

32. (a) $(2, -3, 4), (1, 2, -1), (x, -1, 2)$ are coplanar.

$$\text{So, } \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ x & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(4 - 1) + 3(2 + x) + 4(-1 - 2x) = 0$

$\Rightarrow 6 + 6 + 3x - 4 - 8x = 0$

$\Rightarrow 5x = 8$

$\Rightarrow x = \frac{8}{5}$

33. (d) Unit vector || to the sum of $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $b\hat{i} + 2\hat{j} + 3\hat{k}$ is

$$\vec{r} = \frac{(2+b)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+b)^2 + 36 + 4}}$$

$\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 1$ (given)

$\Rightarrow (2+b) + 6 - 2 = \sqrt{(2+b)^2 + 40}$

$\Rightarrow (2 + b)^2 + 16 + 8(2 + b) = (2 + b)^2 + 40$

$\Rightarrow 8b = 40 - 16 - 16 = 40 - 32$

$\Rightarrow b = 1$

34. (a) $\vec{a} = (2, -1, 1), \vec{b} = (1, 2, -1), \vec{c} = (1, 1, -2)$

Vector in the plane of $\vec{b}$ and $\vec{c}$,

$\vec{r} = m\vec{b} + \vec{c}$

$\vec{r} = (m+1, 2m+1, -m-2)$

Projection of $\vec{r}$ on $\vec{a}$ $= \frac{\vec{r}\cdot\vec{a}}{|\vec{a}|} = \sqrt{\frac{2}{3}}$

$\frac{2(m+1)-(2m+1)-m-2}{\sqrt{4+1+1}} = \pm\sqrt{\frac{2}{3}}$

$2m + 2 - 2m - 1 - m - 2 = \pm 2$

$-m - 1 = \pm 2$

$\Rightarrow m = -3$ or 1

So, $\vec{r} = (-2, -5, -3)$ {For $m = -3$}

$\vec{r} = (2, 3, -3)$ for $m = 1$

So, $\vec{r} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

35. (b) Unit vector $\perp$ to $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{r} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{r} = \frac{-\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{1+4+4}} = \frac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k})$

36. (c) Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a}\cdot\vec{b} = 3 \Rightarrow x + y + z = 3$ ...(i)

$\vec{a}\times\vec{b} = \vec{c}$

$\Rightarrow \hat{i}(z-y) + \hat{j}(x-z) + \hat{k}(y-x) = (0, 1, -1)$

$\Rightarrow z = y, x = 1 + z, y = -1 + x$

$\Rightarrow y = z$ and $x = 1 + z$ Put in eq. (i)

$\Rightarrow 1 + z + z + z + 3 \Rightarrow z = \frac{2}{3}$

$y = \frac{2}{3}$ and $x = \frac{5}{3}$

37. (d) $|\vec{a}| = 4, |\vec{b}| = 4, |\vec{c}| = 5$

$\because \vec{a} \perp (\vec{b}+\vec{c}) \Rightarrow \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = 0;$

$\vec{b} \perp (\vec{c}+\vec{a}) \Rightarrow \vec{b}\cdot\vec{c} + \vec{b}\cdot\vec{a} = 0$

and $\vec{c} \perp (\vec{a}+\vec{b}) \Rightarrow \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} = 0$

$\Rightarrow 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{a}\cdot\vec{a}) = 0$

$|\vec{a}+\vec{b}+\vec{c}|^2 = a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

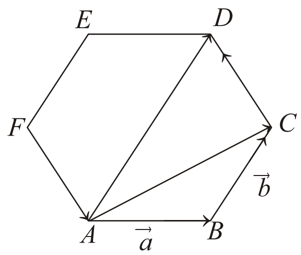
$= 16 + 16 + 25 + 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{57}$

38. (b) Unit vector $\perp$ to $\vec{a}$ and $\vec{b}$ both is $\pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

So, there can be at most two unit vectors $\perp$ to $\vec{a}$ and $\vec{b}$.

39. (b) From diagram



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$

and $\overrightarrow{AD} = 2\overrightarrow{BC} = 2\vec{b}$

then $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$

or $\overrightarrow{CD} = \overrightarrow{AD} - \overrightarrow{AC} = \vec{b} - \vec{a}$

then $\overrightarrow{FA} = -\overrightarrow{CD} = \vec{a} - \vec{b}$

40. (d) $\vec{a} + 2\vec{b} = x\vec{c} \Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = (x+6)\vec{c}$ ...(i)

$\vec{b} + 3\vec{c} = y\vec{a}$

$\Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = (2y+1)\vec{a}$ ...(ii)

Hence, $(x+6)\vec{c} = (2y+1)\vec{a}$

$\Rightarrow \vec{a}$ and $\vec{c}$ are collinear

but given $\vec{a}$ and $\vec{c}$ are non-collinear.

So, $x + 6 = 0$ and $2y + 1 = 0$

$\Rightarrow x = -6$ and $y = -\frac{1}{2}$

So, from eq. (i), $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$

41. (d) $\vec{a} + \vec{b} = x\vec{c} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (x+1)\vec{c}$ ...(i)

$\vec{b} + \vec{c} = y\vec{a} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (y+1)\vec{a}$ ...(ii)

From eqs. (i) and (ii),

$(x+1)\vec{c} = (y+1)\vec{a}$

$\Rightarrow \vec{a}$ and $\vec{c}$ are collinear but given $\vec{a}$ and $\vec{c}$ are non-collinear.

So, $x + 1 = 0$ and $y + 1 = 0$

$\Rightarrow x = -1$ and $y = -1$

So, from eq. (i),

$\vec{a} + \vec{b} = -\vec{c}$

or $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

42. (c) Let $\overrightarrow{OA} = 10\hat{i} + 3\hat{j}, \overrightarrow{OB} = 12\hat{i} - 5\hat{j}$ and

$\overrightarrow{OC} = a\hat{i} + 11\hat{j}$

Then, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\hat{i} - 8\hat{j}$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (a-10)\hat{i} + 8\hat{j}$

Then, $2\hat{i}-8\hat{j}=-1\{-(a-10)\hat{i}-8\hat{j}\}$

{∵ Points are collinear.}

or $2=-(a-10)$ or $a=8$

43. (a) Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=\vec{c}$,

then position vector of centroid

$$\overrightarrow{OG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

So, $\overrightarrow{GA}=\overrightarrow{OA}-\overrightarrow{OG}=\dfrac{2\vec{a}-\vec{b}-\vec{c}}{3}$

$$\overrightarrow{GB}=\overrightarrow{OB}-\overrightarrow{OG}=\frac{2\vec{b}-\vec{a}-\vec{c}}{3}$$

$$\overrightarrow{GC}=\overrightarrow{OC}-\overrightarrow{OG}=\frac{2\vec{c}-\vec{a}-\vec{b}}{3}$$

Then, $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$

44. (b) Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=\vec{c}$

Given $\overrightarrow{AC}=3\overrightarrow{AB}\Rightarrow\vec{c}-\vec{a}=3(\vec{b}-\vec{a})$

or $\vec{c}=3\vec{b}-2\vec{a}$

45. (c) Let the horizontal force be $F_1\hat{i}$ and F_2 be the force inclined at an angle 60° with the vertical so it will makes 90° + 60° = 150° angle with horizontal.

So, $\vec{F_2}=F_2\cos 150°\hat{i}+F_2\sin 150°\hat{j}$

then $\vec{F_1}+\vec{F_2}=P\hat{j}$

or $F_1\hat{i}+F_2\cos 150\hat{i}+F_2\sin 150\hat{j}=P\hat{j}$

$$\left(F_1-F_2\frac{\sqrt{3}}{2}\right)\hat{i}+\frac{F_2}{2}\hat{j}=P\hat{j}$$

$\Rightarrow \dfrac{F_2}{2}=P$ or $F_2=2P$

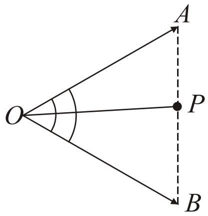
and $F_1-\dfrac{F_2\sqrt{3}}{2}=0$ or $F_1=P\sqrt{3}$

46. (c) Let the force P be long $\hat{i}$, equal to magnitude of resultant force (P) and perpendicular to it, i.e., resultant is along $\hat{j}$ and let other force be $\vec{Q}$

$$P\hat{i}+Q=P\hat{j}\Rightarrow Q=-P\hat{i}+P\hat{j}$$

or $|Q|=\sqrt{P^2+P^2}=P\sqrt{2}$

47. (c) $\overrightarrow{OA}=\hat{i}+3\hat{j}-2\hat{k}$ and $\overrightarrow{OB}=3\hat{i}+\hat{j}-2\hat{k}$



Given, $\overrightarrow{OP}$ bisects $\angle AOB$

∵ $|\overrightarrow{OA}|=|\overrightarrow{OB}|=\sqrt{14}$

and $\angle AOP=\angle BOP$

and OP is common.

∴ P divides the line AB in 1 : 1.

So, $\overrightarrow{OP}=\dfrac{\overrightarrow{OA}+\overrightarrow{OB}}{1+1}=\dfrac{4\hat{i}+4\hat{j}-4\hat{k}}{2}$

$=2(\hat{i}+\hat{j}-\hat{k})$

48. (d) $\vec{a}+2\vec{b}+3\vec{c}=0$ …(i)

$\Rightarrow \vec{a}=-2\vec{b}-3\vec{c}$

or $\vec{a}\times\vec{b}=(-2\vec{b}-3\vec{c})\times\vec{b}=-3(\vec{c}\times\vec{b})$

and $\vec{a}\times\vec{c}=(-2\vec{b}-3\vec{c})\times\vec{c}=-2(\vec{b}\times\vec{c})$

or $\vec{c}\times\vec{a}=2(\vec{b}\times\vec{c})$

So, $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$=-3(\vec{c}\times\vec{b})+\vec{b}\times\vec{c}+2(\vec{b}\times\vec{c})=6(\vec{b}\times\vec{c})$

or $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$=(\vec{a}\times\vec{b})+\dfrac{1}{3}(\vec{a}\times\vec{b})+\dfrac{2}{3}(\vec{a}\times\vec{b})=2(\vec{a}\times\vec{b})$

So, all are correct.

49. (c) Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}, \vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$

Then, given $|\vec{a}|=|\vec{b}|=|\vec{c}|$

$\Rightarrow x^2+y^2+z^2=2$ …(i)

∵ $\vec{c}$ makes an obtuse angle with X-axis.

So, $\vec{c}\cdot\hat{i}<0\Rightarrow x<0$ …(ii)

∵ $\vec{a},\vec{b},\vec{c}$ taken pairwise makes equal angle.

So, $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}=\dfrac{\vec{c}\cdot\vec{a}}{|\vec{c}||\vec{a}|}$

$\Rightarrow \dfrac{1}{2}=\dfrac{y+z}{2}=\dfrac{x+y}{2}$

or $y+z=1$ and $x+y=1$

or $x=z$ and $y=1-x$

From eq. (i), $x^2+(1-x)^2+x^2=2$

or $3x^2-2x-1=0$

$\Rightarrow x=1$ or $x=-\dfrac{1}{3}$

So, $x=-\dfrac{1}{3}$ {From eq. (ii)}

So, $x=-\dfrac{1}{3}, y=\dfrac{4}{3}$ and $z=-\dfrac{1}{3}$

So, $\vec{c}=-\dfrac{1}{3}\hat{i}+\dfrac{4}{3}\hat{j}-\dfrac{1}{3}\hat{k}$

50. (c) Given, $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

and let $|\vec{a}|=|\vec{b}|=|\vec{c}|=p$

Then, $\cos\theta = \dfrac{\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|}$

$= \dfrac{\vec{a}\cdot\vec{a}}{p\cdot\sqrt{a^2+b^2+c^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})}}$

$= \dfrac{p^2}{p\cdot p\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

51. (c) Given $\dfrac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \dfrac{\vec{w}\cdot\vec{u}}{|\vec{u}|}$ and $\vec{v}\cdot\vec{w} = 0$

$\Rightarrow (\vec{v}-\vec{w})\cdot\vec{u} = 0$ ...(i)

Now, $\vec{u}-\vec{v}+\vec{w} = \vec{u}-(\vec{v}-\vec{w})$

$\therefore |\vec{u}-\vec{v}+\vec{w}|^2 = |\vec{u}-(\vec{v}-\vec{w})|^2$

$= |\vec{u}|^2 + |\vec{v}-\vec{w}|^2 - 2\vec{u}\cdot(\vec{v}-\vec{w})$

$= u^2+v^2+w^2-2\vec{v}\cdot\vec{w}-0 = 1+4+9-2(0) = 14$

So, $|\vec{u}-\vec{v}+\vec{w}| = \sqrt{14}$

52. (b) $(2\hat{i}-m\hat{j}+3m\hat{k})\cdot\{(1+m)\hat{i}-2m\hat{j}+\hat{k}\} > 0$

$\Rightarrow 2+2m+2m^2+3m > 0$

or $2m^2+3m+2 > 0$

or $(2m+1)(m+2) > 0$

or $m < -2$ or $m > -\dfrac{1}{2}$

53. (b) Let $\vec{a} = 2p\hat{i}+\hat{j}$

Let $\vec{b}$ be the vector on rotation, then $\vec{b} = (p+1)\hat{i}+\hat{j}$

Then, under rotation about origin $|\vec{a}| = |\vec{b}|$

$\Rightarrow 4p^2+1 = (p+1)^2+1$

or $3p^2-2p-1 = 0$

$(3p+1)(p-1) = 0$

$\Rightarrow p = -\dfrac{1}{3}$ or $p = 1$

54. (c) $(\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b}) = 0$

or $7a^2-15b^2+16\vec{a}\cdot\vec{b} = 0$ ...(i)

and $(\vec{a}-5\vec{b})\cdot(7\vec{a}+3\vec{b}) = 0$

or $7a^2-15b^2-32\vec{a}\cdot\vec{b} = 0$ ...(ii)

From eqs. (i) and (ii)

$48\vec{a}\cdot\vec{b} = 0$ or $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow \vec{a}\perp\vec{b}$

55. (b) $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OD} = \vec{d}$

Given, $|\vec{a}-\vec{d}| = |\vec{b}-\vec{d}| = |\vec{c}-\vec{d}|$

$\Rightarrow |\overrightarrow{DA}| = |\overrightarrow{DB}| = |\overrightarrow{DC}|$

$\Rightarrow$ D is equidistant from vertices A and B and C, so D is the circumcentre of ΔABC.

56. (c) Given, $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OD} = \vec{d}$

Then, $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = 0$

$\Rightarrow \overrightarrow{DB}\cdot\overrightarrow{CB} = 0 \Rightarrow \overrightarrow{DA}\perp\overrightarrow{BC}$

and $(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$

$\Rightarrow \overrightarrow{DB}\cdot\overrightarrow{AC} = 0 \Rightarrow \overrightarrow{AC}\perp\overrightarrow{BD}$

$\Rightarrow D$ is the orthocentre of ΔABC.

57. (a) Adjacent sides $\vec{a} = 3\hat{i}-2\hat{j}+2\hat{k}$

and $\vec{b} = -\hat{i}-2\hat{k}$

So, diagonals are $\vec{d}_1 = \vec{a}+\vec{b} = 2\hat{i}-2\hat{j}$

and $\vec{d}_2 = \vec{a}-\vec{b} = 4\hat{i}-2\hat{j}+4\hat{k}$.

Angle between diagonals,

$\cos\theta = \dfrac{\vec{d}_1\cdot\vec{d}_2}{|\vec{d}_1||\vec{d}_2|} = \dfrac{8+4}{2\sqrt{2}\cdot 6}$

$\cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}$

58. (a) Resultant of two forces

$R^2 = F_1^2+F_2^2+2F_1F_2\cos\theta$

So, $7^2 = P^2+3^2+2P\cdot 3\cos\theta$

or $40 = P^2+6P\cos\theta$ ...(i)

and $19 = P^2+9+2P\cdot 3\cos(\pi-\theta)$

{for reverse direction}

or $10 = P^2-6P\cos\theta$ ...(ii)

From eqs. (i) and (ii),

$50 = 2P^2$

or $P^2 = 25$

i.e., $P = 5$

59. (b) $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix} = -4\hat{j}-3\hat{j}+6\hat{k}$$

So, $|\vec{a}\times\vec{b}| = \sqrt{16+9+36} = \sqrt{61}$

$[\vec{a}\ \vec{b}\ \vec{c}] = (\vec{a}\times\vec{b})\cdot\vec{c} = |\vec{a}\times\vec{b}|\cdot|\vec{c}|\cos\theta$

$= \sqrt{61}\cos\theta = \sqrt{61}\{\max\ \cos\theta = 1\}$

60. (c) Let $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

Then $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = |\vec{a}||\vec{b}\times\vec{c}|\cos\theta$ ...(i)

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 6 \\ 2 & -7 & -10 \end{vmatrix} = 2\hat{i}+2\hat{j}-\hat{k}$$

$|\vec{b}\times\vec{c}| = \sqrt{4+4+1} = 3$

So, from eq. (i),

$[\vec{a}\ \vec{b}\ \vec{c}] = 3\cos\theta$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]$ is max. when $\cos\theta = 1 \Rightarrow \theta = 0$

$\Rightarrow \vec{a}$ is collinear to $\vec{b}\times\vec{c}$.

$\Rightarrow \vec{a} = \lambda(\vec{b}\times\vec{c})$

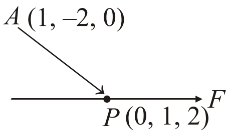
or $|\vec{a}| = |\lambda||\vec{b}\times\vec{c}|$

$1 = \lambda\sqrt{4+4+1}$

or $\lambda = \dfrac{1}{3}$

So, $\vec{a} = \dfrac{1}{3}(\vec{b}\times\vec{c}) = \dfrac{1}{3}(2\hat{i}+2\hat{j}-\hat{k})$

61. (a) $\vec{F} = \hat{i}+2\hat{j}-3\hat{k}+2\hat{i}+3\hat{j}+3\hat{k}-\hat{i}-\hat{j}-\hat{k}$



$\vec{F} = 2\hat{i}+4\hat{j}$

$\vec{r} = \overrightarrow{AP}$

$\vec{r} = -\hat{i}+3\hat{j}+2\hat{k}$

Then, $\overrightarrow{M} = \vec{r}\times\vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 2 \\ 2 & 4 & 0 \end{vmatrix}$$

$= -8\hat{i}+4\hat{j}-10\hat{k}$

62. (c) Diagonals $\vec{d}_1 = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{d}_2 = \hat{i}-3\hat{j}+4\hat{k}$

$\therefore$ Area of parallelogram $= \dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$

$$= \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{matrix}\right\| = \frac{1}{2}|-2\hat{i}-14\hat{j}-10\hat{k}|$$

$= \dfrac{1}{2}\sqrt{4+196+100}$

$= \dfrac{1}{2}\sqrt{300} = 5\sqrt{3}$ sq. units

63. (a) $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a}-\vec{b}| = a^2+b^2-\vec{a}\cdot\vec{b}$

$= 1+1-2ab\cos\theta$

$= 2(1-\cos\theta)$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 4\sin^2\dfrac{\theta}{2} \Rightarrow \sin\dfrac{\theta}{2} = \dfrac{|\vec{a}-\vec{b}|}{2}$

64. (a) ΔABC is an equilateral triangle such that $\overline{OA} = \vec{a}, \overline{OB} = \vec{b}$ and $\overline{OC} = \vec{c}$ and O is orthcentre.

$\Rightarrow \dfrac{\vec{a}+\vec{b}+\vec{c}}{3} = 0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c} = \vec{0}$

65. (a) $\because$ C is the middle point of AB and P is any point outside AB.

Then, from mid point theorem

$\overrightarrow{PC} = \dfrac{\overrightarrow{PA}+\overrightarrow{PB}}{2}$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB} = 2\overrightarrow{PC}$

66. (b) $|\hat{a}| = 1 = |\hat{b}|$

$|\hat{a}+\hat{b}|^2 = |\hat{a}|^2+|\hat{b}|^2+2\hat{a}\cdot\hat{b}$

$= 1+1+2|\hat{a}||\hat{b}|\cos\theta$

$= 2(1+\cos\theta) = 4\cos^2\theta/2$

$\Rightarrow \cos\left(\dfrac{\theta}{2}\right) = \dfrac{|\hat{a}+\hat{b}|}{2}$

67. (d) $a = 3, b = 5, c = 7,\ \vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \vec{a}+\vec{b} = -\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow a^2+b^2+2ab\cos\theta = c^2$

$\Rightarrow 9+25+30\cos\theta = 49$

$\Rightarrow 30\cos\theta = 49-34 = 15$

$\Rightarrow \cos\theta = \dfrac{15}{30} = \dfrac{1}{2} = \cos 60°$

$\Rightarrow \theta = 60° = \dfrac{\pi}{3}$

68. (d) $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$

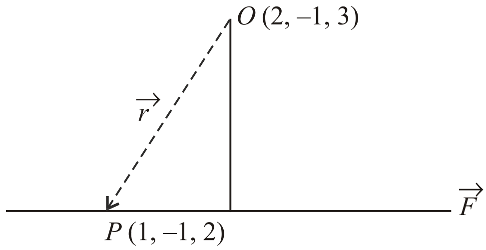
Given, $\vec{a}+\vec{b}+\vec{c} = 0$

$\because |\vec{a}+\vec{b}+\vec{c}|^2 = a^2+b^2+c^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$\Rightarrow 0 = 4+9+16+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\dfrac{29}{2}$

69. (c) Force $\vec{F} = 3\hat{i}+2\hat{j}-4\hat{k}$ is acting at point P(1, –1, 2)



Then, $\vec{r} = \overrightarrow{OP} = (\hat{i}-\hat{j}+2\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})$

$\vec{r} = -\hat{i} - \hat{k}$

Moment of a force about O

$\vec{M} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix} = 2\hat{i} - 7\hat{j} - 2\hat{k}$$

70. (a) $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{k}$

$\because \vec{c} \parallel \vec{a} \Rightarrow \vec{c} = t\vec{a} = t(\hat{i} + \hat{j})$

and $\vec{d} \perp \vec{a} \Rightarrow \vec{d} \cdot \vec{a} = 0$ …(i)

given $\vec{b} = \vec{c} + \vec{d}$

$\Rightarrow \vec{d} = \vec{b} - \vec{c} = 3\hat{i} + 4\hat{k} - t(\hat{i} + \hat{j})$

$\vec{d} = (3-t)\hat{i} - t\hat{j} + 4\hat{k}$

…(ii)

From eq. (i), $\vec{d} \cdot \vec{a} = 0$

$\Rightarrow (3-t) - t = 0 \Rightarrow t = \frac{3}{2}$

So, $\vec{c} = \frac{3}{2}(\hat{i} + \hat{j})$

and $\vec{d} = \frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} + 4\hat{k}$

{From eq. (ii)}

71. (d) From solution of 70,

$\vec{d} = x\hat{i} + y\hat{j} + z\hat{k} = \frac{3}{2}\hat{i} - 3\hat{j} + 4\hat{k}$

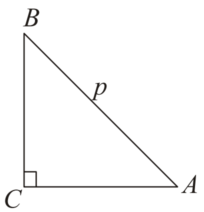
$\Rightarrow x = \frac{3}{2}, y = -\frac{3}{2}$ and $z = 4$

Now, $y - x = -\frac{3}{2} - \frac{3}{2} = -3$ and

$2z - 3 = 2(4) - 3 = 5$

So, neither 1 nor 2 is correct.

72. (c) $\vec{AC} \cdot \vec{BC} = 0$ $\{\because \vec{AC} \perp \vec{BC}\}$



Let C be the origin and position vectors of A and B are $\vec{a}$ and $\vec{b}$.

So, $\vec{CA} = \vec{a}$ and $\vec{CB} = \vec{b}$

Then, $\vec{AB} = \vec{b} - \vec{a}$ and $\vec{a} \cdot \vec{b} = 0$

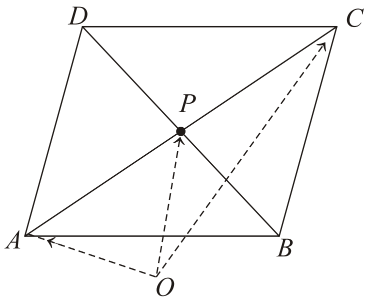
Then, $\vec{AB} \cdot \vec{AC} + \vec{BC} \cdot \vec{BA} + \vec{CA} \cdot \vec{CB}$

$= (\vec{b} - \vec{a}) \cdot (-\vec{a}) + (-\vec{b}) \cdot (\vec{a} - \vec{b}) + 0$

$= a^2 + b^2 = CA^2 + CB^2 = BA^2 = p^2$

73. (d) $\because$ The diagonals of a parallelogram bisect each other. So, P is the middle point of AC and BD both.

$\therefore \vec{OA} + \vec{OC} = 2\vec{OP}$ and $\vec{OB} + \vec{OD} = 2\vec{OP}$



$\Rightarrow \vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OP}$

74. (b) $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$

$\because \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow 2(3) + 3(2) + 4(-\lambda) = 0$

$\Rightarrow 12 - 4\lambda = 0$

$\Rightarrow \lambda = 3$

75. (b) Let the position vector of the vertices of the quadrilateral $ABCD$ are as follows.

$\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$ and $\vec{OD} = \vec{d}$

Then, diagonal $\vec{AC} = \vec{c} - \vec{a}$ and $\vec{BD} = \vec{d} - \vec{b}$

Then, $\vec{BD} + \vec{CA} = (\vec{d} - \vec{b}) - (\vec{c} - \vec{a})$

$= (\vec{a} - \vec{b}) + (\vec{d} - \vec{c}) = \vec{BA} + \vec{CD}$

76. (a) Given $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{b} \cdot (\vec{a} \times \vec{b}) = \vec{c} \cdot \vec{b}$ and $\vec{a} \cdot (\vec{a} \times \vec{b}) = \vec{a} \cdot \vec{c}$

and $\vec{b} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot \vec{a}$

$\Rightarrow [\vec{b}\ \vec{a}\ \vec{b}] = \vec{b} \cdot \vec{c}$ and $[\vec{a}\ \vec{a}\ \vec{b}] = \vec{a} \cdot \vec{c}$

and $[\vec{b}\ \vec{b}\ \vec{c}] = \vec{a} \cdot \vec{b}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs.

Now, again $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{c} \cdot (\vec{a} \times \vec{b}) = \vec{c} \cdot \vec{c}$ and $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot \vec{a}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = |\vec{c}|^2$ and $[\vec{a}\ \vec{b}\ \vec{c}] = |\vec{a}|^2$

$\Rightarrow |\vec{a}| = |\vec{c}|$

and then similarly $|\vec{b}| = 1$.

77. (d) $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + m\hat{j} + n\hat{k}$

$\because \vec{a}, \vec{b}, \vec{c}$ are coplanar vectors.

So, $[\vec{a}, \vec{b}, \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$$

$\Rightarrow 1(3n - 2m) + 1(2n - 2) + 1(2m - 3) = 0$
$\Rightarrow 5n - 5 = 0$
or $n = 1$

Now, $|\vec{c}| = \sqrt{6}$

$\Rightarrow \sqrt{1 + m^2 + n^2} = \sqrt{6}$

or $m^2 = 6 - (1 + n^2)$

$m^2 = 6 - 2 = 4$

$m = \pm 2$

78. (b) Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j}$

$\vec{a} \times \hat{j} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j} = x\hat{k} - z\hat{i}$

$\vec{a} \times \hat{k} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{k} = -x\hat{j} + y\hat{i}$

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$

$= |-y\hat{k} + z\hat{j}|^2 + |x\hat{k} - z\hat{i}|^2 + |-x\hat{j} + y\hat{i}|^2$

$= y^2 + z^2 + x^2 + z^2 + x^2 + y^2$

$= 2\,(x^2 + y^2 + z^2) = 2|\vec{a}|^2$

79. (a) $|\hat{a}| = |\hat{b}| = 1$

$(\hat{a} + \hat{b}) \times (\hat{a} \times \hat{b}) = \hat{a} \times (\hat{a} \times \hat{b}) + \hat{b} \times (\hat{a} \times \hat{b})$

$= (\hat{a} \cdot \hat{b})\hat{a} - (\hat{a} \cdot \hat{a})\hat{b} + (\hat{b} \cdot \hat{b})\hat{a} - (\hat{b} \cdot \hat{a})\hat{b}$

$= (\hat{a} \cdot \hat{b})\hat{a} - \hat{b} + \hat{a} - (\hat{a} \cdot \hat{b})\hat{b}$

$= (\hat{a} - \hat{b})(\hat{a} \cdot \hat{b} - 1)$ …(i)

$\{\because\ (\vec{a} \cdot \vec{b} - 1)$ is scalar quantity.$\}$

$\Rightarrow (\hat{a} + \hat{b}) \times (\hat{a} \times \hat{b}) \parallel (\hat{a} - \hat{b})$

80. (b) $\vec{\alpha} = \hat{i} + 2\hat{j} - \hat{k}, \vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and

$\vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$

$\because\ \vec{\alpha} \perp \vec{\delta}$ and $\vec{\beta} \perp \vec{\delta}$ and $\vec{\delta} \cdot \vec{\gamma} = 10$

So, $\vec{\alpha} \cdot \vec{\delta} = 0 = \vec{\beta} \cdot \vec{\delta}$ and $\vec{\delta} \cdot \vec{\gamma} = 10$

Let $\vec{\delta} = x\hat{i} + y\hat{j} + z\hat{k}$

So, $x + 2y - z = 0$ …(i)

$2x - y + 3z = 0$ …(ii)

and $2x + y + 6z = 10$ …(iii)

On solving eq. (i), (ii) and (iii), we get
$x = -2,\ y = 2$ and $z = 2$

So, $\vec{\delta} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

$\Rightarrow |\vec{\delta}| = \sqrt{(-2)^2 + (2)^2 + (2)^2} = 2\sqrt{3}$ units.